

Use algebraic reasoning to solve the following equations for the given variable. Show all work.

Use your calculator ONLY for equations involving e and *natural log*.

For the trigonometric equations, find solutions $0 \leq \theta \leq 2\pi$

1. $x^6 + 7x^3 = 8$

$u = x^3$

$u^2 + 7u - 8 = 0$

$(u + 8)(u - 1) = 0$

$u = -8 \quad u = 1$

$x^3 = -8 \quad x^3 = 1$

$x = -2 \quad x = 1$

3. $x - 3\sqrt{x} + 2 = 0$

$u = \sqrt{x}$

$u^2 - 3u + 2 = 0$

$(u - 2)(u - 1) = 0$

$u = 2 \quad u = 1$

$\sqrt{x} = 2 \quad \sqrt{x} = 1$

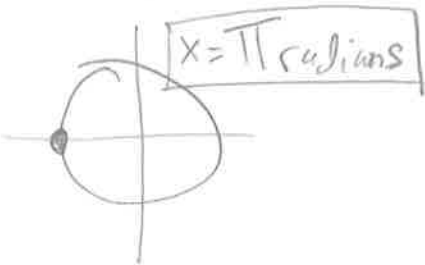
$x = 4 \quad x = 1$

5. $2\cos x + 2 = 0$

$2(\cos x + 1) = 0$

$\cos x + 1 = 0$

$\cos x = -1$



2.

$9x^4 - 3x^2 = 2$

$u = 3x^2$

$u^2 - u - 2 = 0$

$(u - 2)(u + 1) = 0$

$u = 2 \quad u = -1$

$3x^2 = 2$

$3x^2 = -1$

$x^2 = \frac{2}{3}$

$x^2 = -\frac{1}{3}$

$x \approx 0.816$

no solution

4. $e^{4x} + 35 = 12e^{2x}$

$u = e^{2x}$

$u^2 + 35 = 12u$

$u^2 - 12u + 35 = 0$

$(u - 7)(u - 5) = 0$

$u = 7 \quad u = 5$

$e^{2x} = 7 \quad e^{2x} = 5$

$\ln 7 = 2x \quad \ln 5 = 2x$

$\frac{\ln 7}{2} = x \approx 0.973$

$\frac{\ln 5}{2} = x \approx 0.805$

6. $n^{10} - 5n^6 + 4n^2 = 0$

$n^2(n^8 - 5n^4 + 4) = 0$

$u = n^4$

$\sqrt{u}(u^2 - 5u + 4) = 0$

$\sqrt{u}(u - 4)(u - 1) = 0$

$u = 0 \quad u = 4 \quad u = 1$

$n^4 = 0 \quad n^4 = 4 \quad n^4 = 1$

$n = 0, n \approx 1.414, n = 1$

$$7. (m+5)^4 - 5 = m$$

$$(m+5)^4 = m+5$$

$$u = m+5$$

$$u^4 = u$$

$$u^4 - u = 0$$

$$u(u^3 - 1) = 0$$

$$u=0 \quad u^3=1$$

$$u=1$$

$$m+5=0 \quad m+5=1$$

$$m = -5$$

$$m = -4$$

$$9. (\sqrt[3]{x})^2 + 2 \cdot \sqrt[3]{x} = 3$$

$$u = \sqrt[3]{x}$$

$$u^2 + 2u - 3 = 0$$

$$(u+3)(u-1) = 0$$

$$u = -3 \quad u = 1$$

$$(\sqrt[3]{x} = -3)^3 \quad (\sqrt[3]{x} = 1)^3$$

$$x = -27, x = 1$$

$$11. (\log_5(x-1))^2 - \log_5(x-1) = 2$$

$$u = \log_5(x-1)$$

$$u^2 - u - 2 = 0$$

$$(u-2)(u+1) = 0$$

$$u = 2 \quad u = -1$$

$$\log_5(x+1) = 2 \quad \log_5(x+1) = -1$$

$$5^2 = x+1$$

$$5^{-1} = x+1$$

$$x = 24$$

$$x = -\frac{4}{5}$$

$$8. x^3 + 2x^2 - x - 2 = 0$$

$$x^3 - x + 2x^2 - 2 = 0$$

$$x(x^2-1) + 2(x^2-1) = 0$$

$$u = x^2 - 1$$

$$xu + 2u = 0$$

$$u(x+2) = 0$$

$$u=0 \quad x=-2$$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$\rightarrow x = \pm 1 \quad x = -2$$

$$10. \sin^3 \theta - \sin^2 \theta = 2 \sin \theta$$

$$u = \sin \theta$$

$$u^3 - u^2 - 2u = 0$$

$$u(u^2 - u - 2) = 0$$

$$u(u-2)(u+1) = 0$$

$$u=0 \quad u=2 \quad u=-1$$

$$\sin \theta = 0$$

~~$\sin \theta = 2$~~
Not possible

$$\sin \theta = -1$$



$$\theta = 0, \pi$$

$$\theta = \frac{3\pi}{2}$$

$$12. (x^3 - 3x^2)(4x - 12) = 0$$

$$x^2(x-3) - 4(x-3) = 0$$

$$u = x-3$$

$$x^2u - 4u = 0$$

$$u(x^2 - 4) = 0$$

$$u(x+2)(x-2) = 0$$

$$u=0 \quad x=2 \quad x=2$$

$$x-3=0$$

$$x = 3, x = -2, x = 2$$